Stability Sensitivity Studies for Synthesis of Aeroelastic Systems

Yi Lu* and V. R. Murthy†
Syracuse University, Syracuse, New York

Introduction

STABILITY sensitivity analyses play an important role in the optimal design of systems with dynamic constraints. Adelman and Haftka[1] recently published an excellent review paper on the topic. Wittrick[2] derived the equations for the derivatives of eigenvalues for self-adjoint systems, and Fox and Kapoor[3] presented the expressions for the derivatives of eigenvectors for self-adjoint systems using two different formulations. The first formulation involves only the eigenvalue and the eigenvector under consideration, and the second formulation involves all the eigenvalues and eigenvectors. Rogers[4] and Plaut and Huseyin[5] independently extended the second formulation of Ref. 3 to non-self-adjoint systems. Garg,[6] Rudisill,[7] and Nelson[8] presented alternate formulations to compute the derivative of an eigenvector of a non-self-adjoint system requiring only the knowledge of the corresponding eigenvalue and its right and left eigenvectors. Taylor and Kane[9] extended the basic formulation of Ref. 3 to the quadratic eigenvalue problems. Rudisill and Chu[10] presented two numerical methods to determine the derivatives of eigenvalues and eigenvectors of constant coefficient systems, which is a special case of the present formulation.

The relations given in Refs. 3–9 are not directly applicable to the dynamic aeroelastic stability problems because these are double eigenvalue (ν and ω) problems. In general, these eigenvalues do not appear in polynomial form in the equations of motion. Cardani and Mantegazza[11] presented a formulation to determine the derivatives of flutter eigensolutions. Bindolino and Mantegazza[12] used an adjoint problem to improve the procedure of the approach presented in Ref. 11. These two references basically provide the procedures to determine the derivatives of eigenvalues and eigenvectors with respect to the system parameters. The present synoptic provides a direct method to perform sensitivity analyses for discrete aeroelastic systems in the Laplace transform domain. The method can give the derivatives of the eigenvalues as well as the eigenvectors simultaneously.

Formulation and Application

The equation of motion of an aeroelastic system in the Laplace transform domain can be written as

$$[B(s)]\{\bar{\eta}\} = \{0\} \tag{1}$$

The eigenvalue s_k and eigenvector $\{p_k\}$ ($k = 1,2,\ldots,N$) satisfy

$$[B(s_k)]\{p_k\} = \{0\} \tag{2}$$

Received Sept. 29, 1988; synoptic received Aug. 21, 1989. Full paper available from National Technical Information Service, Springfield, VA 22151, by title, at the standard price (available upon request).

*Graduate Teaching Assistant, Department of Mechanical and Aerospace Engineering. Member AIAA.

†Associate Professor, Department of Mechanical and Aerospace Engineering. Member AIAA.

The eigenvector $\{p_k\}$ can be normalized as

$$\{p_k\}^T\{p_k\}/2 = 1 \tag{3}$$

Taking the variation for Eqs. (2) and (3) yields

$$\delta[B(s_k)]\{p_k\} + [B(s_k)]\delta\{p_k\} = \{0\} \tag{4}$$

$$\{p_k\}^T\delta\{p_k\} = \{0\} \tag{5}$$

$$\delta[B(s_k)] = \delta_1[B(s_k)] + \delta_2[B(s_k)] \tag{6}$$

where $\delta_1[B(s_k)]$ is the variation of $[B(s_k)]$ due to the variation of the system parameters in $[B(s)]$ at $s = s_k$

$$\delta_2[B(s_k)] = \left.\frac{\partial[B]}{\partial s}\right|_{s=s_k} \delta s_k \tag{7}$$

Substituting Eqs. (6) and (7) into Eq. (4) yields

$$\left.\frac{\partial[B]}{\partial s}\right|_{s=s_k} \{p_k\}\delta s_k + \delta_1[B(s_k)]\{p_k\} + [B(s_k)]\delta\{p_k\} = \{0\} \tag{8}$$

Combining Eqs. (8) and (5) yields

$$\begin{bmatrix} \left.\frac{\partial[B]}{\partial s}\right|_{s=s_k}\{p_k\} & [B(s_k)] \\ 0 & \{p_k\}^T \end{bmatrix} \begin{Bmatrix} \delta s_k \\ \delta\{p_k\} \end{Bmatrix} = \begin{Bmatrix} -\delta[B]\{p_k\} \\ 0 \end{Bmatrix} \tag{9}$$

where $\delta_1[B]$ has been replaced by $\delta[B]$ for simplicity.

$$\begin{Bmatrix} \dfrac{\partial s_k}{\partial b_{ij}} \\ \dfrac{\partial\{p_k\}}{\partial b_{ij}} \end{Bmatrix} = -[H]_k\{I\}p_{jk} \tag{10}$$

where

$$[H]_k = \begin{bmatrix} \left.\frac{\partial[B]}{\partial s}\right|_{s_k}\{p_k\} & [B(s_k)] \\ 0 & \{p_k\}^T \end{bmatrix}^{-1}$$

where $\{I\}$ is the column vector with $I_\ell = 0$ ($\ell \neq i$) and $I_i = 1$.

If the system matrix $[B]$ is a function of a parameter P then

Table 1 Derivatives of eigenvalues

		S_1	S_2	S_3
		Direct analytical approach		
β_{pc}		$-3.4520 - 3.3030i$	$3.8190 + 2.8320i$	$-0.2920 - 0.1773i$
EI_y		$-0.08 + 6.87i$	$-0.042 - 0.5391i$	$0.045 + 85.6i$
EI_z		$0.06085 - 2.9732i$	$0.04162 + 4.3603i$	$(2.76 + 16.82i)10^{-3}$
GJ		$0.075 - 0.01652i$	$-0.3973 + 0.02492i$	$0.1402 + 0.052i$
γ		$-0.3880 - 0.0902i$	$-0.1474 - 0.004i$	$-0.4135 + 0.223i$
Ω	$10^{-2}\times$	$-22.5200 - 8.3610i$	$-9.8550 + 10.7300i$	$-24.9700 + 0.3288i$
e_0	$10^{-1}\times$	$-3.0080 - 1.7580i$	$-0.0666 + 0.3241i$	$-0.1647 - 0.1907i$
β	$10^{-2}\times$	$26.8600 + 52.3600i$	$-65.8400 - 66.6200i$	$-50.9600 - 9.3400i$
C^*		$-1.9060 - 0.5956i$	$-0.7729 + 0.0680i$	$-2.0710 - 0.1263i$
		Finite difference method		
β_{pc}		$-3.3967 - 2.562i$	$3.7708 + 2.474i$	$-0.30141 - 0.2861i$
EI_y		$-0.0681 + 7.35i$	$-0.0218 - 0.3731i$	$0.036 + 64.78i$
EI_z		$0.06222 - 2.392i$	$0.331 + 4.0565i$	$(3.6 + 12.6i)10^{-3}$
GJ		$0.01 - 0.1988i$	$-0.734 + 0.3793i$	$0.1611 + 0.04935i$
γ		$-0.3828 - 0.1422i$	$-0.15245 - 0.008716i$	$-0.40845 - 0.0370i$
Ω	$10^{-2}\times$	$-27.2416 - 8.8686i$	$-6.334 + 11.6966i$	$-26.577 + 1.671i$
e_0	$10^{-1}\times$	$-3.171 - 0.95272i$	$-0.08374 + 0.4692i$	$-0.2341 - 0.1120i$
β	$10^{-2}\times$	$25.4562 + 55.624i$	$-67.749 - 72.765i$	$-57.5735 - 11.253i$

the derivatives of eigendata are given by

$$\begin{Bmatrix} \dfrac{\partial s_k}{\partial P} \\ \dfrac{\partial \{p_k\}}{\partial P} \end{Bmatrix} = -[H]_k \begin{Bmatrix} \dfrac{\partial [B]}{\partial P}\{p_k\} \\ 0 \end{Bmatrix} \tag{11}$$

The properties of the blade considered for the calculation of numerical results are taken from Ref. 13. The first five free vibration mode shapes are used in the stability and sensitivity analysis; these include three flapping modes, one chordwise mode, and one torsion mode. The natural frequencies associated with these modes about the initial state and the nonlinear trim state are computed using the transfer matrix method.[14]

The analytically derived derivatives and finite difference derivatives of the eigenvalues with respect to the several system parameters are computed and presented in Table 1. These results validate the formulation and the numerical calculations. The following conclusions can be drawn from the results presented in Table 1.

1) Precone angle β_{pc}: The precone angle affects mainly the first flap and lead-lag motion. Since the real part of $\partial s_2/\partial\beta_{pc} \approx 3.8$, the precone angle has a strong unstable effect on the second mode, that is, lead-lag mode.

2) Pretwist β: When the pretwist is increased, the lead-lag mode stability is improved $[Re(\partial s_2/\partial\beta) \approx -0.65]$ and the first flap mode has a small instability. β also improves the stability of second and third flapping modes.

3) Aerodynamic center offset from the elastic axis e_0: This parameter has a large effect on the torsion mode, and instability can occur in this mode with increase of e_0.

4) Aerodynamic parameter $\gamma = 6k_4R/m$: The stability of all modes is improved with increasing γ, which is an obvious effect.

5) Derivatives with respect to stiffness properties: As expected, flapping stiffness significantly affects the flapwise bending frequencies s_1 and s_3 and similarly the chordwise stiffness significantly affects the in-plane frequency s_2.

6) Unsteady aerodynamics (Theodorsen's function C^*): The stability is improved when the unsteady aerodynamic effects are included due to the inertia and lag effects in the unsteady aerodynamics. The stabilizing effect is more predominant in the flapping modes.

Conclusions

A direct and comprehensive formulation is presented to perform the sensitivity analyses of discrete aeroelastic systems in the s domain. The significant features of the method are 1) the unsteady aerodynamic forces can be considered without the augmented state variables, 2) the derivatives of eigenvalues and eigenvectors can be obtained simultaneously, 3) the formulation involves only the eigenvalue and eigenvector under consideration, and 4) in addition to the derivatives of eigendata with respect to system parameters, the relative effect of the system matrix elements can be obtained directly, which is very useful to adjust the parameters to get a system of satisfied dynamic characteristics. Finally, the method is applied to perform the sensitivity analysis of dynamic characteristics of cantilever rotor blades in hover.

References

[1]Adelman, H. M., and Haftka, R. T., "Sensitivity Analysis of Discrete Structural Systems," *AIAA Journal*, Vol. 24, May 1986, pp. 823–832.

[2]Wittrick, W. H., "Rates of Change of Eigenvalues with Reference to Buckling and Vibration Problems," *Journal of the Royal Aeronautical Society*, Vol. 66, No. 621, Sept. 1962, pp. 590–591.

[3]Fox, R. L., and Kapoor, M. P., "Rates of Change of Eigenvalues and Eigenvectors," *AIAA Journal*, Vol. 6, Dec. 1968, pp. 2426–2429.

[4]Rogers, L. C., "Derivatives of Eigenvalues and Eigenvectors," *AIAA Journal*, Vol. 8, May 1970, pp. 943–944.

[5]Plaut, R. H., and Huseyin, K., "Derivatives of Eigenvalues and Eigenvectors in Non-Self-Adjoint Systems," *AIAA Journal*, Vol. 11, Feb. 1973, pp. 250–251.

[6]Garg, S., "Derivatives of Eigensolutions for a General Matrix," *AIAA Journal*, Vol. 11, Aug. 1973, pp. 1191–1194.

[7]Rudisill, C. S., "Derivatives of Eigenvalues and Eigenvectors of a General Matrix," *AIAA Journal*, Vol. 12, May 1974, pp. 721–722.

[8]Nelson, R. B., "Simplified Calculation of Eigenvector Derivatives," *AIAA Journal*, Vol. 14, Sept. 1976, pp. 1201–1205.

[9]Taylor, D. L., and Kane, T. R., "Multiparameter Quadratic Eigenvalue Problems," *Journal of Applied Mechanics*, Vol. 42, No. 2, 1975, pp. 478–483.

[10]Rudisill, C. S., and Chu, Y. Y., "Numerical Methods for Evaluating the Derivatives of Eigenvalues and Eigenvectors," *AIAA Journal*, Vol. 13, June 1975, pp. 834–837.

[11]Cardani, C., and Mantegazza, P., "Calculation of Eigenvalue and Eigenvector Derivatives for Algebraic Flutter and Divergence Eigenproblems," *AIAA Journal,* Vol. 17, April 1979, pp. 408–412.

[12]Bindolino, P., and Mantegazza, P., "Aeroelastic Derivatives as a Sensitivity Analysis of Nonlinear Equations," *AIAA Journal*, Vol. 25, Aug. 1987, pp. 1145–1146.

[13]Hodges, D. H., and Ormiston, R. A., "Stability of Elastic Bending and Torsion of Uniform Cantilever Rotor Blades in Hover with Variable Structural Coupling," NASA TN-D-8192, April 1976.

[14]Shultz, L. A., and Murthy, V. R., "Dynamic Analysis of Multiple-Load-Path Blades by the Transfer Matrix Method," *Journal of Sound and Vibration* (submitted for publication).